

CBCS SCHEME

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18MAT31

Third Semester B.E. Degree Examination, Jan./Feb. 2023 Transform Calculus, Fourier Series and Numerical Techniques

Time: 3 hrs.

Max. Marks: 100

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

- 1 a. Find the Laplace transform of:
- $(3t + 4)^2 + 5^t$
 - $e^{-t} \cos^2 3t$
 - $\frac{\cos at - \cos bt}{t}$
- (10 Marks)
- b. Given $f(t) = \begin{cases} E, & 0 < t < a/2 \\ -E, & a/2 < t < a \end{cases}$ where $f(t+a) = f(t)$, show that $L[f(t)] = \frac{E}{S} \tanh(as/4)$.
(05 Marks)
- c. Employ Laplace transform to solve the equation: $y'' + 5y' + 6y = 5e^{2t}$, taking $y(0) = 2$, $y'(0) = 1$.
(05 Marks)

OR

- 2 a. Find the Inverse Laplace transform of:
- $\frac{(s+2)^2}{s^6}$
 - $\frac{s+1}{s^2+6s+9}$
 - $\frac{3s+2}{s^2-s-2}$
- (10 Marks)
- b. Express $f(t) = \begin{cases} 1, & 0 < t \leq 1 \\ t, & 1 < t \leq 2 \\ t^2, & t > 2 \end{cases}$ in terms Heaviside's unit step function and hence find its Laplace transform.
(05 Marks)
- c. Find the Laplace transform of $\frac{s}{(s^2+a^2)^2}$ using convolution theorem.
(05 Marks)

Module-2

- 3 a. Find the Fourier series expansion of $f(x) = x - x^2$ in $-\pi \leq x \leq \pi$. Hence deduce that $\frac{x^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots$
(07 Marks)
- b. Find the half-range cosine series of $f(x) = 2x-1$ in the interval $0 < x < 1$.
(06 Marks)
- c. Determine the constant term and the first cosine and sine terms of the Fourier series expansion of y from the following data:

x°	0	45	90	135	180	225	270	315
y	2	3/2	1	1/2	0	1/2	1	3/2

(07 Marks)

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and/or equations written eg. 42+8=50, will be treated as malpractice.

OR

- 4 a. Obtain the Fourier series of $f(x) = |x|$ in $(-l, l)$. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$. (07 Marks)

- b. Find the sine half range series of $f(x) = \begin{cases} \frac{1}{4} - x & \text{in } 0 < x < \frac{1}{2} \\ x - \frac{3}{4} & \text{in } \frac{1}{2} < x < 1 \end{cases}$ (06 Marks)

- c. The following table gives the variations of a periodic current A over a certain period T:

t(sec)	0	T/6	T/3	T/2	2T/3	5T/6	T
A(amp)	1.98	1.30	1.05	1.30	-0.88	-0.25	1.98

Show that there is a constant part of 0.75amp. in the current A, and obtain the amplitude of the first harmonic. (07 Marks)

Module-3

- 5 a. If $f(x) = \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| \geq 1 \end{cases}$ find the Fourier transform of $f(x)$ and hence find the value of $\int_0^{\infty} \frac{x \cos x - \sin x}{x^3} dx$. (07 Marks)
- b. Find the Fourier sine and cosine transform of $f(x) = e^{-\alpha x}$, $\alpha > 0$. (06 Marks)
- c. Solve $u_{n+2} + 4u_{n+1} + 3u_n = 3^n$, given $u_0 = 0$, $u_1 = 1$ by using z-transform. (07 Marks)

OR

- 6 a. Find the Fourier sine transform of $f(x) = e^{-|x|}$ and hence evaluate $\int_0^{\infty} \frac{x \sin mx}{1+x^2} dx$, $m > 0$. (07 Marks)
- b. Find the Z-transform of $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$.
- c. Find the inverse Z-transform of $\frac{3z^2 + 2z}{(5z-1)(5z+2)}$. (06 Marks)
- (07 Marks)

Module-4

- 7 a. Solve $\frac{dy}{dx} = x - y^2$, $y(0) = 1$ using Taylor's series method considering upto fourth degree terms and find the value of $y(0.1)$. (07 Marks)
- b. Using Runge-Kutta method of fourth order, find $y(0.2)$ for the equation $\frac{dy}{dx} = \frac{y-x}{y+x}$, $y(0) = 1$ taking $h = 0.2$. (06 Marks)
- c. Apply Milne's method to compute $y(1.4)$ correct to four decimal places given $\frac{dy}{dx} = x^2 + \frac{y}{2}$ and the data: $y(1) = 2$, $y(1.1) = 2.2156$, $y(1.2) = 2.4649$, $y(1.3) = 2.7514$.

(07 Marks)

OR

- 8 a. Using modified Euler's method find $y(20.2)$ given that $\frac{dy}{dx} = \log_{10}\left(\frac{x}{y}\right)$ with $y(20) = 5$ taking $h = 0.2$. (07 Marks)
- b. Use Fourth order Runge-Kutta method to compute $y(1.1)$ given that $\frac{dy}{dx} = xy^{1/3}$, $y(1) = 1$. (06 Marks)
- c. If $\frac{dy}{dx} = 2e^x - y$, $y(0) = 2$, $y(0.1) = 2.010$, $y(0.2) = 2.040$ and $y(0.3) = 2.090$, find $y(0.4)$ using Adams – Bashforth predictor-corrector method. (07 Marks)

Module-5

- 9 a. Given $\frac{d^2y}{dx^2} - x^2 \frac{dy}{dx} - 2xy = 1$, $y(0) = 1$, $y'(0) = 0$, evaluate $y(0.1)$ using Runge-Kutta method of 4th order. (07 Marks)
- b. Find the external of the functional $\int_{x_1}^{x_2} (y'^2 - y^2 + 2y \sec x) dx$. (06 Marks)
- c. Derive Euler's equation in the standard form:

$$\frac{\partial f}{\partial y} - \frac{d}{dx} \left(\frac{\partial f}{\partial y'} \right) = 0.$$
 (07 Marks)

OR

- 10 a. Apply Milne's method to compute $y(0.8)$ given that $\frac{d^2y}{dx^2} = 1 - 2y \frac{dy}{dx}$ and the following table of initial values:

x	0	0.2	0.4	0.6
y	0	0.02	0.0795	0.1762
y'	0	0.1996	0.3937	0.5689

- (07 Marks)
- b. Find the external of the functional $\int_0^{\pi/2} (y'^2 - y'^2 - 2y \sin x) dx$ under the end conditions $y(0) = 0$, $y(\pi/2) = 0$. (06 Marks)
- c. Prove that the geodesics on a plane are straight lines. (07 Marks)
